Lab III

Projectile Motion

1 Introduction

In this lab we will look at the motion of a projectile in two dimensions. When one talks about a 'projectile', the implication is that we give an object an initial velocity, after which it moves only under the influence of gravity, much like a ball being thrown.

While the overall motion of a projectile may look complicated, projectile motion can be broken up into accelerated motion in the vertical direction (freefall, as in Lab II) and constant velocity motion in a horizontal direction.

The fundamental basis of projectile motion is that we have a constant acceleration in a particular direction ('downwards'). Whenever there is a constant acceleration, we always have the freedom to choose coordinates so that one of our axes (the y-axis, for example) is in the direction of the acceleration. As a result, the acceleration is only along one axis, with zero acceleration (constant velocity) along the other axes.

We can then decompose the motion of the projectile into horizontal and vertical components, and these components can be treated independently of one another.

This technique of decomposing motion into components that can be treated separately and independently is a powerful one, and greatly simplifies many problems involving motion.

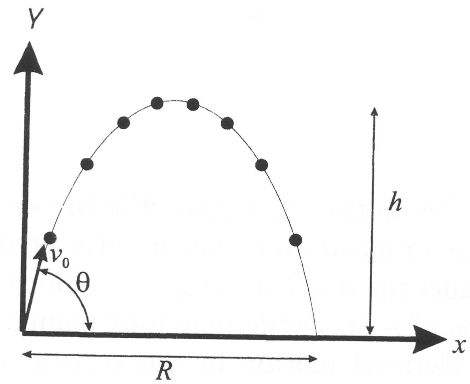
2 Theory

First we choose our x-axis to be horizontal, and our y-axis to be vertical pointing upward (see Fig. III.1). Gravity will give our object an acceleration g downward (along the negative y-axis), while the acceleration along the x-axis is zero:

$$a_x = 0 \quad \text{(III.1a)}$$
$$a_y = -g \quad \text{(III.1b)}$$

where g is the acceleration of gravity. Gravity is the only force we'll consider in this situation; the effect of air drag is small and (mostly) negligible.

Figure III.1: Projectile motion trajectory, where the dots are positions of the projectile at equal time intervals.



As a result, we can write equations of motion:

$$x(t) = x_0 + v_{0x}t \quad \text{(III.2a)}$$
$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2 \quad \text{(III.2b)}$$

where x_0 and y_0 are the values of x and y at $t = 0$, v_{0x} and v_{0y} are the velocities at $t = 0$, and g is the acceleration of gravity.

Taking the time derivatives of Eqn. III.2, we get velocities:

$$v_x(t) = v_{0x} \quad \text{(III.3a)}$$
$$v_y(t) = v_{0y} - gt \quad \text{(III.3b)}$$

or, expressed as a vector:

$$\mathbf{v}(t) = v_x\mathbf{i} + v_y\mathbf{j} \quad \text{(III.4a)}$$
$$= v_{0x}\mathbf{i} + (v_{0y} - gt)\mathbf{j}. \quad \text{(III.4b)}$$

The point where the projectile reaches maximum height h, is known as the **apex** of its motion. The apex is the point at which the vertical component of the velocity, v_y, is zero. So to find h, we first find the time t_{apex} when $v_y(t_{apex}) = 0$. From Eqn. III.3b,

$$v_y(t_{apex}) = v_{0y} - g t_{apex} = 0 \quad \text{(III.5a)}$$

$$t_{apex} = \frac{v_{0y}}{g}, \quad \text{(III.5b)}$$

substituting t_{apex} for t in Eqn. III.2b, and taking the definition of h as the height above where the projectile was launched (y_0):

$$h = y(t_{apex}) - y_0 = v_{0y} t_{apex} - \frac{1}{2} g {t_{apex}}^2 \quad \text{(III.6a)}$$

$$= v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2 \quad \text{(III.6b)}$$

$$= \frac{v_{0y}^2}{2g}. \quad \text{(III.6c)}$$

Note that h is independent of any horizontal velocity: it only depends on the vertical component of initial velocity and the vertical acceleration.

So far we have only dealt with the velocity in terms of its components v_x and v_y, but it's often useful to instead talk about the magnitude of the velocity (speed) $v = |\mathbf{v}|$ and the angle θ from horizontal (see Fig. III.1). A little trigonometry gives us the relationship between these quantities:

$$v_x = v \cos\theta \quad \text{(III.7a)}$$

$$v_y = v \sin\theta \quad \text{(III.7b)}$$

and $v = \sqrt{v_x^2 + v_y^2}$ by the Pythagorean Theorem. Expressing the velocity in terms of magnitude and angle is most useful when talking about the initial velocity $\mathbf{v}_0$, in which case the initial speed v_0 and angle θ can be used to get the initial velocity components v_{0x} and v_{0y}.

The shape of the projectile's trajectory can be obtained by solving Eqn. III.2a for the time t in terms of x, then substituting into Eqn. III.2b, so that we get y in terms of x:

$$y = y_0 + \frac{v_{0y}}{v_{0x}}(x - x_0) - \frac{g}{2v_{0x}^2}(x - x_0)^2. \quad \text{(III.8)}$$

If we pick an origin so that $x_0 = 0$, then Eqn. III.8 is much simpler, and it clearly describes a parabola in terms of a quadratic function of x with constant coefficients.

Keep these coefficients in mind when fitting the trajectory of a projectile, since (for example) if one measures $y(x)$ and fits the measurements with a quadratic function of x and obtains:

$$y(x) = (0.3\,\mathrm{m}) + (1.2)x - (2.1\,\mathrm{m}^{-1})x^2 \tag{III.9}$$

it tells us that $y_0 = 0.3\,\mathrm{m}$, $v_{0y}/v_{0x} = 1.2$ and $g/(2v_{0x}) = 2.1\,\mathrm{m}^{-1}$. Compare the terms in Eqn. III.8 (with $x_0 = 0$) with the terms in Eqn. III.9 to see how the fit of the trajectory gives us values for the coefficients in Eqn. III.8.

Name: ____________________ **Sec/Group:** _____ **Date:** _____

3 Pre-Lab

1. Use Eqns. III.2 to derive the equation for the trajectory of the projectile, Eqn. III.8.

2. Derive the equation for the horizontal range R for a projectile to return to the height at which it was launched (see Fig. III.1), from the initial velocity, angle, and the acceleration of gravity. Calculate the range of a projectile launched with $|\mathbf{v}| = 4.7\,\text{m/s}$ and $\theta = 60°$ from horizontal.

3. **Projectile Motion Simulation**
   See `http://www.physics.drexel.edu/labs/` for simulation.

   (a) Pick an angle (θ) and velocity (v_0) for the simulation so the range is about 1 m and the height is about 0.6 m. Enter the values you used in Table. III.1.

Name: ____________________ **Sec/Group:** _____ **Date:** _____

(b) Run the simulation and analyze the trajectory. Get the x and y positions from the simulation. (Note that the images of the projectile are taken at intervals of 1/30 sec). Print a copy of the analysis page showing the trajectory and your data points.

(c) From the x positions, verify that for any interval $\Delta x/\Delta t$ is equal to the initial horizontal velocity $|\mathbf{v}_0|\cos\theta$ by doing a linear fit of x as a function of t. Pick two points on the trajectory to use for calculating Δx and Δt to show this. Print out and hand in your plot.

(d) Do a quadratic fit of the y positions as a function of time t. Print out and hand in your plot.

(e) Plot the y values as a function of x and fit with a quadratic. Print out and hand in your plot.

(f) Enter the fit coefficients you found in the previous steps in Table III.1, and explain the meaning of the coefficients (*i.e.*, the constant term is y_0) by comparing with Eqn. III.2b and Eqn. III.8. Also fill in the correct units for each of the fit coefficients.

(g) Use the range equation that you derived in Prelab question 2 to calculate the range R, and compare to the simulation. If there's a big difference, you better re-check your calculations in question 2!

4. Turn in your prelab answers, Table III.1, and your plots.

Name: ____________ **Sec/Group:** ____ **Date:** ____

Table III.1: PRE-LAB Projectile Motion Simulation Data

$\lvert \mathbf{v}_0 \rvert$	____	[m/s]
θ :	____	[deg]

x motion

Δx	____	[m]
Δt	____	[s]
$v_x = \frac{\Delta x}{\Delta t}$	____	[m/s]
$v_{0x} = \lvert \mathbf{v}_0 \rvert \cos\theta$	____	[m/s]

y motion $y(t) = c_0 + c_1 t + c_2 t^2$

	Fit		Meaning of Coefficient
c_0 :	____	[ ]	____
c_1 :	____	[ ]	____
c_2 :	____	[ ]	____

v_{0y}	____	[m/s]
a_y	____	[m/s^2]

Trajectory $y(x) = d_0 + d_1 x + d_2 x^2$

	Fit		Meaning of Coefficient
d_0 :	____	[ ]	____
d_1 :	____	[ ]	____
d_2 :	____	[ ]	____

Range

Calculated: ____ [m] Measured: ____ [m]

4 Equipment

Figure III.2 shows the launcher that you will use for this lab. Note that the base of the launcher needs to be held securely (preferably clamped to a lab bench) when a ball is launched, so that direction of the launcher is unchanged when the trigger-cord is pulled. **CAUTION: keep clear of the barrel of the launcher when it is loaded, and make sure that the area 'downrange' is clear. Warn anyone nearby to watch out and keep clear before firing the launcher. Remember that the ball may bounce off the floor, wall, or other obstacles and go further than anticipated.**

Figure III.2: Projectile launcher

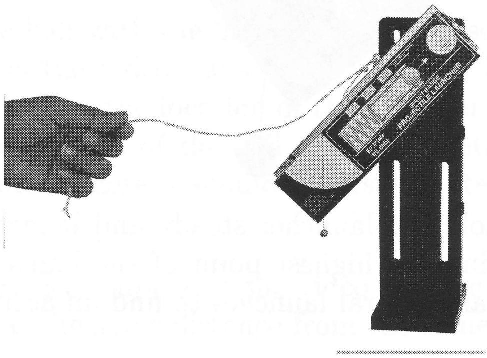


Figure III.3 shows a close-up of the angle setting for the projectile launcher. Make sure that the thread is free to move, and let the hanging weight settle to get an accurate angle.

5 Procedure

5.1 Altitude, Velocity and Range

1. Adjust the launcher so that it is pointing vertically ($\theta = 90°$), and place it on the floor near a wall.

2. Starting from the height of the launcher, hold a meter stick against the wall.

3. Insert a ball into the launcher, pushing it only to the first 'stop' (lowest velocity setting). Keep clear of the launcher opening when it is loaded!

9. Launch a ball with the medium velocity setting. Is your prediction correct? Measure the horizontal range, and enter it in Table 6.

5.2 Trajectory

The video camera should already be set up pointing toward a wall with a dark background. The video frame rate should be set to 30 Hz. See Appendix A for how to capture and analyze video. Before launching any balls, make note of the limits of the video image of the wall.

1. Set the launcher for 75° from horizontal.

2. Put the launcher on the floor, close to and pointing parallel to the wall.

3. Try launching the ball with the 'medium' velocity setting. Do you get enough of the trajectory in the video frame? Try with the 'high' velocity setting also. You may need to adjust the location of the launcher since you want to capture the rise, apex and descent of the ball. Put the container for catching the ball in position (calculate where it should be first). Enter the launch conditions in Table III.3.

4. When the launcher is set up, start the video capture with a meter stick held in the field of view, at the same distance from the camera as the trajectory of the ball.

5. Then launch the ball while capturing. Save the video clip you captured.

6. Calibrate the image scale of the video from the frames containing the meter stick.

7. Measure a series of (x,y) positions for the ball, stepping between video frames, enter the values in Table III.3, as well as the time. Take $t = 0$ at the first frame of your measurements.

8. Plot your x positions as a function of time, and fit with a straight line (see Appendix B); enter the slope (v_{0x}) and intercept (x_0) in Table III.4. Calculate v_{0x} from the launch angle and velocity, as well as the percentage difference between the calculated and measured v_{0x}. Turn in your x *vs.* t plot with your lab.

9. Plot your y positions as a function of time, and fit with a quadratic (polynomial of degree=2), and enter the fit coefficients (y_0, v_{0y} and $-g/2$) in Table III.4; turn in your plot with your lab report.

10. Plot your y positions as a function of x position and fit with a quadratic. Enter the fit coefficients in Table III.4.

Name: ________________ **Sec/Group:** _____ **Date:** _____

6 Data

Table III.2: Launch velocity and range

launcher setting	height [m]	v_0 [m/s]
1	______	______
2	______	______
3	______	______

R(calc) for $\theta = 60°$ ______ [m]

R(meas) for $\theta = 60°$ ______ [m]

Name: ____________ **Sec/Group:** ____ **Date:** ____

Table III.3: Trajectory video capture data. If you have this data in a spreadsheet, just print out the spreadsheet and turn in with your lab.

Video Frame #	t [s]	x [m]	y [m]
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Name: ____________________ **Sec/Group:** _____ **Date:** _____

Table III.4: Projectile motion results summary.

x *vs.* t slope (v_{0x}):	____________	[m/s]
x *vs.* t intercept (x_0):	____________	[m]
$\|\mathbf{v}_0\| \cos\theta = v_{0x}$ (calc):	____________	[m/s]
v_{0x} (meas-calc) %difference:	____________	%

$y(t) = c_0 + c_1 t + c_2 t^2$

c_0 (y_0):	____________	[m]
c_1 (v_{0y}):	____________	[m/s]
c_2 ($-g/2$):	____________	[m/s^2]
$g = -2c_2$:	____________	[m/s^2]

$y(x) = d_0 + d_1 x + d_2 x^2$

d_0:	____________	[ ]
d_1:	____________	[ ]
d_2:	____________	[ ]

Name: ______________ **Sec/Group:** ____ **Date:** ____

7 Conclusions

1. How does the measured range compared to the range you predicted? Determine the percentage difference. If you find that the difference is significant, suggest some possible causes for the difference.

2. Use the results of the fits for x *vs.* t and y *vs.* t to calculate what the coefficients of y *vs.* x should be, and compare to the coefficients from your fit.

3. Does your data show acceleration along the x axis (indicate the data or plot on which you base your conclusion)? What effects could cause you to see an acceleration along x?

4. How does the vertical acceleration in your measurements compare with the 'standard' value of $9.80\,\mathrm{m/s^2}$? Calculate the percentage difference.